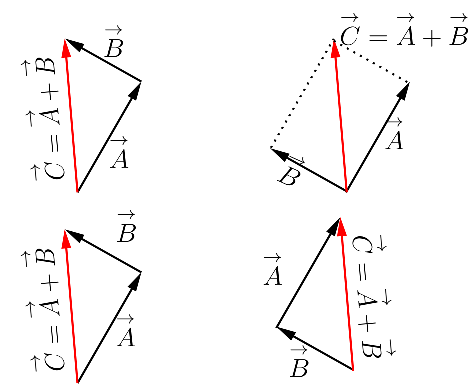
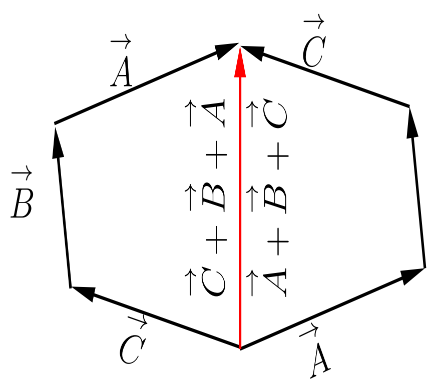
Vectors

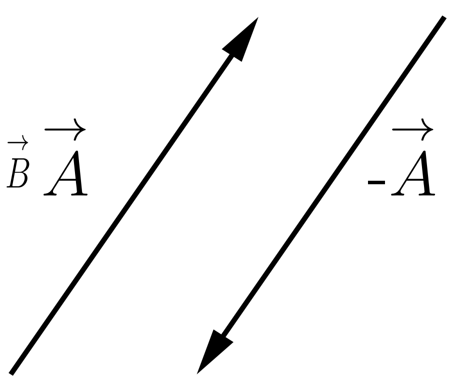
Vector is defined as a set of order pairs in $\mathbb{R}^n$.
Common representations are

$$
\begin{aligned}
\vec{a} &= a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \\
\vec{a} &= \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}^T \\
\vec{a} &= |a|\angle\theta
\end{aligned}
$$

Vector Addition



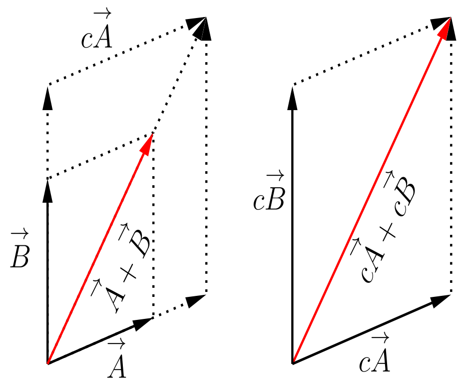




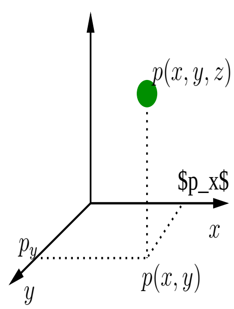
- use head-to-tail rule (good visualization, but not always useful)
- simply add the corresponding terms.

Associate Law	$(A+B)+C = A+(B+C)$
Existence of -ve Vectors	$A+(-A) = -A+A = 0$
Commutative Law	$A+B = B+A$
Distributive Law	$m(A+B) = mA+nB$
	$(m+n)A = mA+nA$

Properties of vector algebra



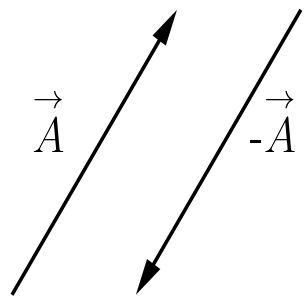
The position vector can be decomposed into sub-components as illustrated



Orthogonal Basis

a set of perpendicular unit vectors forms the standard basis. For example

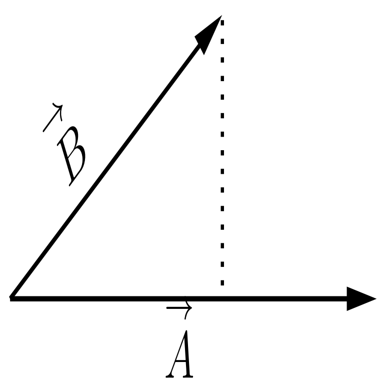
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Vector Multiplication

$$\begin{aligned}
\vec{a} \cdot \vec{b} &= \begin{bmatrix} a_x & a_y & a_z \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \\
&= |a||b| \cos\theta \qquad (1)
\end{aligned}$$

where $g_0(t)$ and $\tilde{n}(t)$ are produced by the signal and noise components present in $r(t)$. A simple way to describe the requirement of output signal component $g_0(t)$ be considerably greater than the output noise component $\tilde{n}(t)$ is to have a filter which makes the instantaneous power of desired signal $g_0(t)$ measured at time $t{=}T_s$ as large as possible compared with the average power of the output noise $\tilde{n}(t)$. This is equivalent to maximizing the peak pulse SNR, defined as



Several Examples of Dot Product in real life

Work Force into Displacement

$$W = \vec{F} \cdot \vec{d}$$

Electric Flux Electric Field Strength into Normal Area

$$\phi = \vec{E} \cdot d\vec{A}$$

Electric Potential Electric Field Strength into Displacement

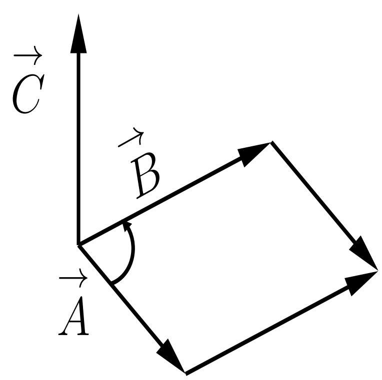
$$V = \vec{E} \cdot d\vec{s}$$

0.1 Vector Product

The cross product or vector product is defined as

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{bmatrix} \hat{i} & \hat{j} & \hat{i} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{bmatrix} \\ &= |a||b| \sin\theta \ \hat{n} \end{aligned} \tag{2}$$

The graphical illustration of the process is defined as



The magnitude of the resultant force is equal to the area of parallelogram of the Please insert the table of left handed axis as well. Several examples of Cross Product in real life:
Torque Force into Displacement

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Angular Momentum

$$\vec{l} = \vec{r} \times \vec{p}$$

Lorentz Force

$$\vec{F} = q\vec{v} \times \vec{B}$$

Properties of vector algebra

Distributive Law	$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$
	$(m+n)\vec{A} = m\vec{A} + n\vec{A}$
null vectors or $\parallel$	$\vec{A} \times \vec{B} = 0$
Commutative property is not valid	$\vec{A} \times \vec{B} = -(\vec{A} \times \vec{B})$

1 Triple Product

Dot and Cross multiplication of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ may produce meaningful products such as a triple products such as $\vec{A}(\vec{B} \cdot \vec{C})$, $\vec{A} \cdot (\vec{B} \times \vec{C})$ and $\vec{A} \times (\vec{B} \times \vec{C})$ some important considerations are as follows:

1. in general $(A \cdot B)C \neq A(B \cdot C)$.
2. $A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$ =volume of parallel-piped with sides A, B and C.
3. generally $A \times (B \times C) \neq (A \times B) \times C$.
4. cross product in terms of dot product

$$\begin{aligned} A \times (B \times C) &= (A \cdot C)B - (A \cdot B)C \\ (A \times B) \times C &= (A \cdot C)B - (B \cdot C)A \end{aligned}$$

Thinking questions

- Find a vector which is perpendicular to vector A:$2\hat{i} - 3\hat{j} + 4\hat{k}$ and B:$-2\hat{i} + \hat{j} - 3\hat{k}$. Is this solution unique?
- Find a vector which is perpendicular to vector A:$-\hat{i} + 2\hat{j} - 2\hat{k}$ and Is this solution unique?
- Are the two vectors parallel A:$-2\hat{i} + \hat{j} - 3\hat{k}$, B:$2\hat{i} - \hat{j} + 3\hat{k}$.
- What is the magnitude and phase of the resultant vector with A:$-\hat{i} + \hat{j} - \hat{k}$ and B:$\hat{i} - \hat{j} + \hat{k}$.

- We have vector
- What is the magnitude and phase of the following vector A $-2\hat{i} + \hat{j} - 3\hat{k}$.
- Show that $A \times B = -B \times A$.
- What is the angle between the following vectors A:$[2\ \ -2\ 1]^T$ and B :$[1\ \ -2\ \ -1]^T$.